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ABSTRACT

Several different mathematical methods for optimization of instruction are presented and compared. A way in which one of these methods might be applied to the measurement and avoidance of inequality in educational opportunity and performance is presented. Finally, it is suggested that computer-assisted instruction might provide a particularly appropriate frame within which these methods might be employed. (RH)

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COMPUTER-ASSISTED INSTRUCTION AND EQUALITY IN EDUCATIONAL ACHIEVEMENT[1]

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This paper extends ideas presented earlier by Jamison, Fletcher, Suppes, and Atkinson (1973), and applies them more explicitly to the optimization of instruction and concomitantly to the evaluation of educational inputs. The version of the paper presented today is preliminary in that the applications of inequality aversion outlined will be extended and explicitly formalized in a version we expect to submit for publication early this summer.[2] It will become apparent that our comments are equally applicable to the optimization of computer-assisted instruction (CAI), to curriculum evaluation, and to the evaluation of educational inputs; the emphasis in this version is on CAI.

There is a current view that what goes on in schools has little effect on the achievement of students. This view received considerable support from the Coleman Report (Coleman, Campbell, Hobson, McPartland, Mood, Weinfeld, & York, 1966) and from Jensen (1969). Coleman concluded that factors within the schools seem to effect achievement much less than do factors outside the schools; these somewhat disheartening conclusions have been subject to rigorous debate since their initial publication, and Jamison, Suppes, and Wells (1973) provide a review of the relevant literature.

Our CAI work, however, has led us to more optimistic conclusions concerning the potential capability of the schools to affect scholastic

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performance. We have found strong and consistent achievement gains by students when they are given CAI over a reasonable fraction of a school year. As Bowles and Levin (1968) pointed out: "The findings of the Report are particularly inappropriate for assessing the likely effects of radical changes in the level and compositions of resources devoted to schooling because the range of variation in most school inputs in this sample is much more limited than the range of policy measures currently under discussion." Many evaluations of CAI provide detailed information about the output effects of a much broader variety of school inputs than the Coleman Report considered.

CAI can be used or abused. Used properly, it is an extremely effective pedagogical tool and presents a serious possibility for the improvement of education. A review of successful CAI projects is not within the scope of this paper. Vinsonhaler and Bass (1972) and Jamison, Suppes, and Wells (1972) present useful reviews of CAI evaluation literature, and we suspect there are other good reviews either available or in preparation.

In addition to the evaluation of CAI, the effort to optimize CAI as an educational input seems to be increasingly prominent. We should note that the terms 'maximization' and 'optimization' are not synonymous. Optimization, at least as we use it, refers to the simultaneous maximization of marginal utilities under constraints that may be interdependent and/or in conflict. Recent work reported by Atkinson and Paulson (1972), Chant and Atkinson (1973), and Laubsch (1970) indicate an increasing interest in the optimization of instruction.

As Atkinson (1972) points out, the derivation of an optimal strategy requires that the problem be stated in a form amenable to decision-theoretic

analysis. Four elements are required prior to the derivation of an optimal instructional strategy:

1. A model of the learning process.
2. Specification of admissible instructional actions.
3. Specification of instructional objectives.
4. A measurement scale that permits costs to be assigned to each of the instructional actions and payoffs to the achievement of instructional objectives.

In this paper, we assume that adequate models of the learning process required by element 1 exist. This assumption is not as cavalier as it may seem. Already, Lorton (1973) has applied incremental and all-or-none models to CAI in spelling, and Laubsch (1970) has applied the random-trial increments model to CAI for foreign language vocabulary.

We assume that element 2 exists as a repertoire of educational inputs. This element is critical in determining the effectiveness of a decision-theory analysis; varying the set of actions from which the decision-maker is free to choose changes the decision problem, even though the other elements remain the same.

We also assume element 3 exists. It is important to distinguish between element 3 and the second half of element 4. Element 3 merely provides that the set of educational outcomes under consideration can be explicated and listed. The second half of element 4 provides a weighting function that can be applied to element 3; it insures that the relative importance of the educational outputs in element 3 can be specified as a ratio scale.

We refer the interested reader to Levin (1970, 1973) for some important work emphasizing the first half of element 4 which may be described as allocative efficiency. Our emphasis is on one aspect of the second half of element 4, explicating the equality of desired educational outcomes.

Having reduced the problem of optimal decision making in educational policy to (perhaps) manageable proportions, we will discuss equality of educational outcome in general terms and show how it can be applied to evaluation and optimization of instruction.[3]

Equality

Our remarks draw illustrative data from two CAI programs: an initial reading program for grades K-3 described by Atkinson and Fletcher (1972), and an elementary school arithmetic program described by Suppes and Morningstar (1969). Evidence that these programs have a positive effect on educational outcomes is presented by Fletcher and Atkinson (1972) for the reading program and by Suppes and Morningstar for the arithmetic program.

Gini coefficients. We first use a traditional measure of inequality, the Gini coefficient, to examine inequality in achievement gains. Consider a group of students who have taken an achievement test; each student will have achieved some score on the test, and there will be a total score obtained by summing all the individual scores. We can then ask what fraction of the total score was obtained by the 10 percent of students doing most poorly on the test, what fraction was obtained by the 20 percent of students doing most poorly, etc. For that matter, we can plot fraction of total score earned by the bottom x percent of students as a function of x.

These concepts may be expressed more formally in the notation of Levine and Singer (1970) as follows. Let $N(u)$ be the achievement-score density function. Then $N(u)du$ represents the number of individuals scoring between u and $u + du$. The total number of students, N, and their average score, A, are given by:

$$N = \int_0^\infty N(u)du, \qquad \text{and}$$

$$A = \frac{1}{N}\int_0^\infty uN(u)du \, .$$

The fraction of students scoring a or less is given by

$$f(a) = \frac{1}{N}\int_0^a N(u)du \, ,$$

and the fraction of the total score obtained by students scoring a or less is

$$g(a) = \frac{\int_0^a uN(u)du}{NA} \, .$$

We can, then, plot $g(a)$ as a function of $f(a)$ for all a with respect to a particular educational outcome. If there is perfectly equal distribution of achievement, the resulting curve, called a Lorenz curve in econometric literature, is a 45° line. The more the obtained Lorenz curve differs from a 45° line--the more it 'sags'--the more unequal is the distribution of achievement. Illustrative Lorenz curves are plotted in Figure 1.

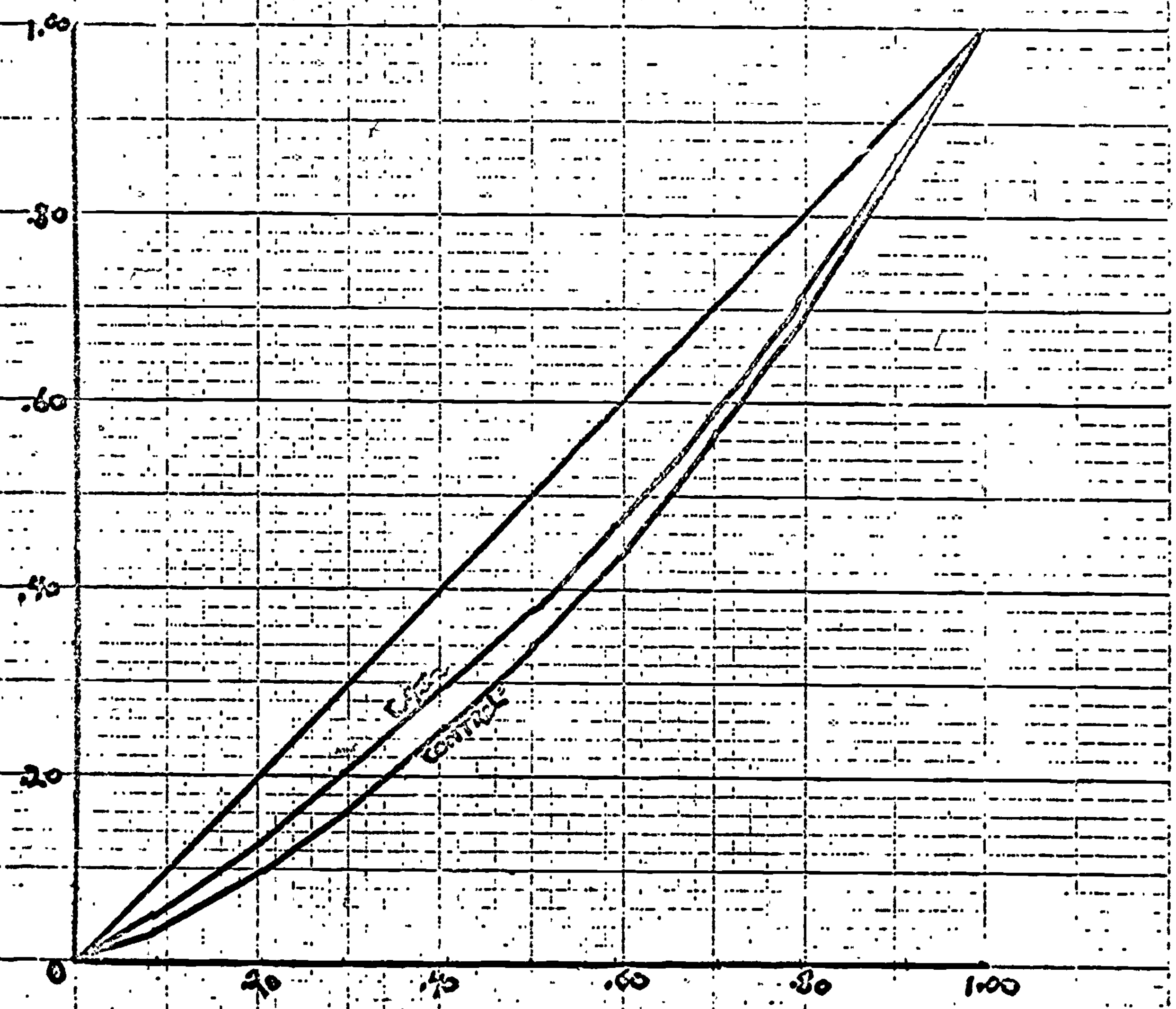

Figure 1 - Typical Lorenz Curves: Distribution of Reading Posttest Achievement (COOP)

The Gini coefficient is an aggregate measure of inequality that is defined as the ratio of the area between g(a) and the 45° line to the area between the 45° line and the abscissa. If the Gini coefficient is zero the distribution of achievement is completely uniform; the larger the Gini coefficient, the more unequal the distribution.

We used Gini coefficients to access the extent to which the reading and arithmetic CAI programs were inequality reducing. Table 1 displays Gini coefficients for CAI and control groups for the three reading posttests used by Fletcher and Atkinson. The subjects in this study were 44 matched pairs of first grade students. The three posttests were the Stanford Achievement Test (SAT), the California Cooperative Primary Reading Test (COOP), and an instrument (DF) designed by the project to test the precise objectives of the CAI reading curriculum. The CAI and control groups are comparable because they were carefully matched by pretest achievement prior to the CAI treatment.

Table 2 displays Gini coefficients for six grades of about 100 students each in Mississippi. For Table 2, we computed Gini coefficients for the distribution of achievement in the CAI and control groups before and after the arithmetic CAI was used. For each group at each grade level we present Gini coefficients for the pretest, for the posttest, and for the difference between the two. This information is given for both the CAI group and the control group. In the final column of Table 2 the difference between columns 3 and 6 is shown; if this difference is positive it indicates that there was a greater reduction in inequality in the CAI group than in the control group.

Presumably, statistical statements could be made about the distribution of Gini coefficients and/or their differences, but we want to limit our discussion of them. Gini coefficients are fairly well established as measures

Table 1

Gini Coefficients for Reading Achievement Posttests[a]

	CAI	Control	Control-CAI
SAT	.134	.174	.040
COOP	.183	.266	.083
DF	.068	.152	.084

[a]Due to careful matching of CAI and control groups by pretest achievement, pretest Gini coefficients are not shown.

Table 2

Gini Coefficients for Control Groups and Experimental Groups Given Arithmetic CAI

	CAI			Control			(CAI Pre-Post) - (Control Pre-Post)
	PRE	POST	PRE-POST	PRE	POST	PRE-POST	
Grade 1[a]	.057	.067	-.010	.037	.062	-.025	.015
2	.064	.039	.025	.055	.050	.005	.020
3	.016	.032	-.016	.035	.038	-.003	-.013
4	.080	.053	.027	.084	.065	.019	.008
5	.095	.070	.025	.078	.079	-.001	.026
6	.068	.077	-.009	.078	.084	-.006	-.003

[a] Gini coefficients are computed from Stanford Achievement Test, Computation subscale, grade placements.

of inequality, but as Anthony Atkinson (1970) has pointed out, they have a number of shortcomings, most notable of which is that they are not purely empirical measures but contain an underlying value judgment concerning what constitutes inequality. Further, Newbery (1970) demonstrated that it is impossible to explicate this value judgment by means of any additive utility function. Therefore, we turn to two value explicit measures of inequality.

Value Explicit Measures of Inequality

Use of the value explicit measures of Gini coefficients implies that achievement test scores should be measured on a ratio scale (i.e., the achievement measure must be unique up to multiplication by a positive constant). If, for example, achievement measures were only unique up to a positive linear transformation, the Gini coefficient could be made arbitrarily small by adding an arbitrarily large amount to each individual's achievement test score. Our assumption that achievement is measured on a ratio scale is quite strong; on the other hand, a ratio scale is essentially implicit in the assumption that one test score is better than another if and only if the number of problems correct on the one test is greater than the number correct on the other.

We take Anthony Atkinson's suggestion, and consider the measure W to be the overall utility of an educational treatment. We define W in terms of a distribution of achievement scores, N(u), as

$$W = \int_0^{\underline{u}} U(u)\ N(u)du \ ,$$

where $\underline{u}$ is the maximum posttreatment measure attainable and U(u) is increasing

and concave. $U(u)$ is, in effect, a weighting function that defines the optimal distribution of posttreatment achievement. The extra requirement of concavity on $U(u)$ implies that the optimal distribution of posttreatment achievement is inequality averting. More precisely, there is a level of achievement, u_e, that is lower than the average level of achievement in the population under consideration such that if everyone in the population had exactly u_e posttreatment achievement the overall utility accruing from the educational treatment would remain constant at W. Further, if $\bar{u}$ is the average level of posttreatment achievement, a reliable measure of inequality, I, is given by

$$I = 1 - \frac{u_e}{\bar{u}} .$$

The lower I is, the more equal is the distribution of achievement. The measure I ranges between 0 for complete equality and 1 for complete inequality and indicates, in effect, by what percentage total achievement could be reduced to obtain the same level of W if the achievement level were equally distributed.

In order to apply the measure I we consider two classes of functions of U. The first of these was suggested by Atkinson and has the property of "constant relative inequality aversion" which simply means that multiplying all achievement levels in the distributions by a positive constant does not alter the measure I of inequality. If there is constant relative inequality aversion it is known from the theory of risk aversion that $U(u)$ must have the following form:

$$U(u) = a + b\,\frac{u^{1-\epsilon}}{1-\epsilon} \quad \text{if} \quad \epsilon \neq 1, \text{ and}$$

$$U(u) = \ln(u) \quad \text{if} \quad \epsilon = 1 .$$

Another possibility that Atkinson considers is that of constant absolute inequality aversion, which means that adding a constant to each achievement level in the distribution does not alter the measure of inequality. A theorem of Pfanzagl (1959) can be used to show that if there is constant absolute inequality aversion, then $U(u)$ must have one of the following two forms:

$$U(u) = au + b \text{ , or}$$

$$U(u) = a\lambda^{u} + b \text{ .}$$

Strict concavity implies the latter of these two and that $0 < \lambda < 1$.

We have then two families of utility functions, one indexed by ϵ and the other by λ. These families include a large number of qualitatively important alternatives for U. In Figure 2 $U(u)$ is shown for several values of ϵ, and in Figure 3 $U(u)$ is shown for several values of λ. Since transforming the functions depicted in Figures 2 and 3 by a positive linear transformation does not affect the measure $\hat{I}$, the height and location of the functions in those two figures is arbitrary.

It is clear that the measure I will vary with ϵ or λ for any fixed distribution of achievement. In Figure 3 we have constrained $U(u)$ to pass through 0 and 1 for all values of λ implying that $U(u) = (1-\lambda^{u})/(1-\lambda)$. For λ very close to 1 inequality is close to 0; as λ gets smaller and smaller, then inequality will get larger for any fixed distribution. The way in which I varies with ϵ is just the opposite; low values of ϵ give a low measure of inequality whereas large values of ϵ give large values for I.

One value of indexing a measure of inequality by some parameter (such as ϵ or λ) that describes the degree of inequality aversion is that achievement

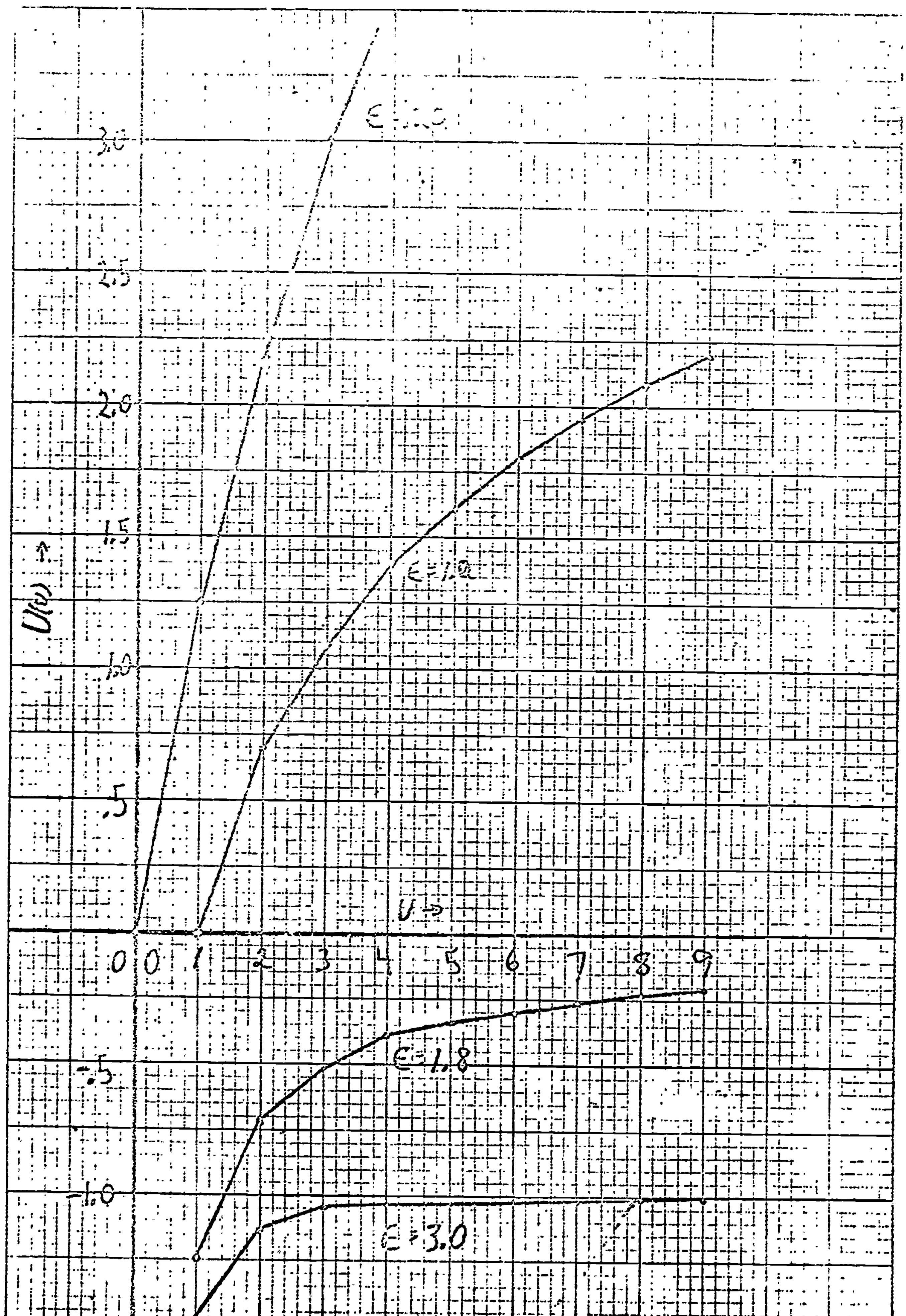

Figure 2 - U(u) for several values of ε.

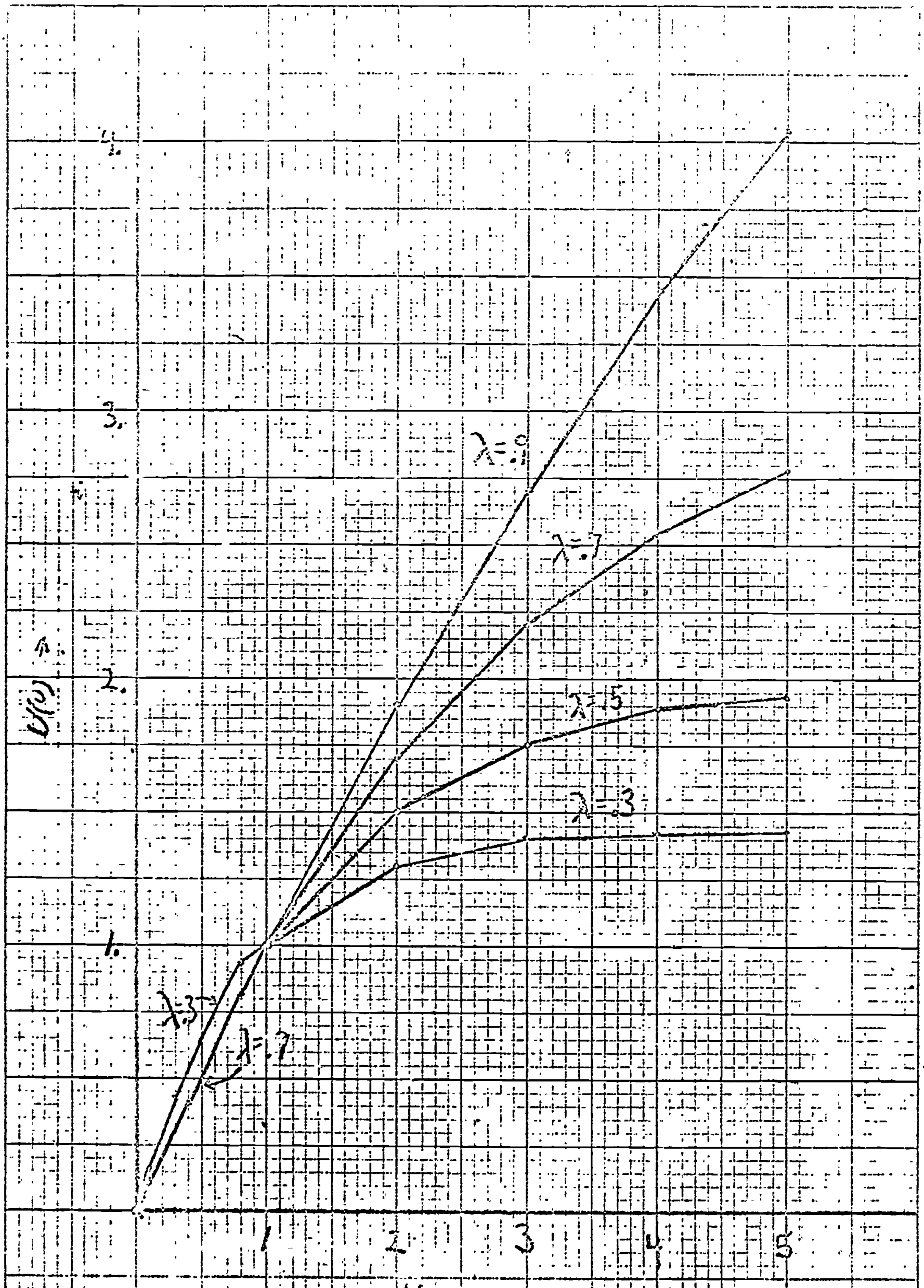

Figure 3 - U(u) for several values of λ.

in some groups may be judged to be more equal for some values of the parameter than for others. Table 3 includes reversals as a function of ϵ under the assumption of constant relative inequality aversion, and Table 4 includes some reversals as a function of λ. From Tables 3 and 4 it can be seen that the parameters (ϵ and λ) explicate value judgments for any posttreatment distribution of achievement.

Applications

Optimization. A very natural application of these measures of inequality is in the optimization of instruction. Procedures that are indicated by such an application are not limited to CAI, but CAI appears to be the most facile medium for them. In any case, a reasonable next step is to design patterns of CAI presentation that are optimal for some utility function U maximized subject to such constraints as the distribution of prior achievement in the student population, the total number of terminal hours available, and/or a production function that relates CAI time or number of CAI sessions to gains in measured achievement.

Elaborate and complex mathematical models are not needed for these applications. Suppes, Fletcher, and Zanotti (1973) applied a simple linear model in setting grade placement goals in CAI. This model was of the form

$$\widehat{GGP}_i = a_0 + a_1 S_i^{1/3}$$

where $\widehat{GGP}_i$ was the goal grade placement for student i, S_i was the number of CAI sessions for student i, and a_0 and a_1 were parameters of the model. The range in standard error for this model over the population of students considered was about .02-.04 years in grade placement; despite its simplicity

Table 3

CAI Inequality Reduction: Constant Relative Inequality

Under Arithmetic CAI[a]

	ϵ							
Student Group	.20	.60	1.0	1.4	1.8	2.2	2.6	3.0
Grade 1	.001	.002	.004	.005	.006	.007	.007	.007
2	.004	.012	.020	.030	.041	.054	.068	.084
3	-.002	-.005	-.008	-.012	-.015	-.019	-.024	-.029
4	.002	.005	.009	.014	.020	.028	.038	.050
5	.005	.012	.019	.023	.026	.027	.025	.022
6	.000	-.002	-.003	-.004	-.006	-.007	-.009	-.010

[a]The numbers shown in the table are $I_A - I_B$ as a function of ϵ. I_A is the difference in inequality between CAI and control after treatment (i.e., on the posttest) and I_B is the difference before treatment. If the difference is greater after treatment than before, CAI is inequality-reducing.

Table 4

CAI Inequality Reduction: Constant Absolute Inequality Aversion Under Arithmetic CAI[a]

	λ							
Student Group	.90	.80	.70	.60	.50	.40	.30	.20
Grade 1	-.001	-.005	-.009	-.011	-.013	-.005	.011	.030
2	.010	.041	.090	.127	.146	.148	.139	.120
3	-.131	-.180	-.237	-.297	-.331	-.331	-.300	-.246
4	-.013	.016	.050	.054	.044	.033	.024	.017
5	.048	.006	-.010	-.007	.000	.004	.009	.016
6	-.083	-.108	-.098	-.078	-.060	-.046	-.037	-.030

[a]The numbers shown in the table are $I_A - I_B$ as a function of λ. I_A is the difference in inequality between CAI and control after treatment (i.e., on the posttest) and I_B is the difference before treatment. If the difference is greater after treatment than before, CAI is inequality-reducing.

the model was quite precise. This model could be easily used to maximize U subject to constraints on the number of CAI sessions that could be assigned within a student population. Such an application would represent a significant improvement over previous work in that the value judgments underlying the notion of optimization would be explicit in the parameters of the utility function.

Evaluation of educational inputs. We believe that a more important application of these explications of utility is to the evaluation of educational inputs in general. There is currently some discussion of a 'meritocracy' preserved by inequality in educational inputs (Stein, 1971). By forcing an explication of the inequality averting properties of these inputs in terms of their probable outcomes, the achievement of equality in educational opportunity can be made far more probable. In any case, we would be disappointed to see indiscriminent application of educational inputs without an effort to explicate the value judgments that underlie their allocation.

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Footnotes

[1]This research was supported by National Science Foundation Grant NSF-443X and by National Aeronautics and Space Administration Grant NGR-05-020-547.

[2]We have a deadline of 1 July 1973.

[3]Originally, we planned to discuss equality of outcome in compensatory education. The restriction of our remarks to compensatory education now appears unnecessary.